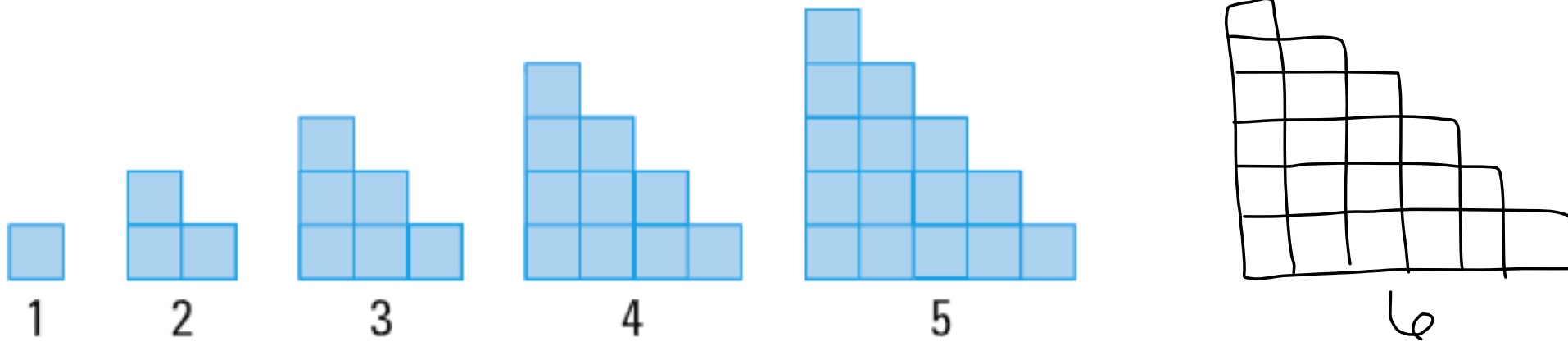# Chapter 1
Basics of Geometry

Section 1
Patterns and Inductive Reasoning

GOAL 1: Finding and Describing Patterns

- Study patterns discovered throughout history

- Learn to recognize and describe patterns of your own

- Make accurate predictions based on patterns

Example 1: Describing a Visual Pattern

Sketch the next figure in the pattern.

Example 2: Describing a Number Pattern

Describe a pattern in the sequence of numbers. Predict the next number.

a. 1, 4, 16, 64, …

x by 4

256

b. -5, -2, 4, 13, …

adding consecutive multiples of 3

25

GOAL 2: Using Inductive Reasoning

Three stages:

1) Look for a pattern

2) Make a conjecture

3) Verify the conjecture

Example 3: Making a Conjecture

Complete the conjecture.

Conjecture: The sum of the first *n* odd positive integers is n^2.

$$
\begin{array}{c|l}
n & \\
\hline
2 & 1 + 3 = 4 \\
3 & 1 + 3 + 5 = 9 \\
4 & 1 + 3 + 5 + 7 = 16 \\
\end{array}
$$

What is a counterexample?

Shows/proves something to be false

Example 4: Finding a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: For all real numbers x, the expression x^2 is greater than or equal to x.

x	$x^2 \geq x$
2	$2^2 \geq 2 \rightarrow 4 \geq 2$ T
3	$3^2 \geq 3 \rightarrow 9 \geq 3$ T
-2	$(-2)^2 \geq -2 \rightarrow 4 \geq -2$ T
0	$0^2 \geq 0 \rightarrow 0 \geq 0$ T
1/2	$(1/2)^2 \geq 1/2 \rightarrow 1/4 \geq 1/2$ F

Example 5: Examining an Unproven Conjecture

In the early 1700s a Prussian mathematician named Goldbach noticed many even numbers greater than 2 can be written as the sum of two primes.

Specific Cases:

$4 = 2 + 2; 6 = 3 + 3; 8 = 3 + 5; 10 = 3 + 7; 12 = 5 + 7; 14 = 3 + 11; 16 = 3 + 13$

Conjecture: Every even number greater than 2 can be written as the sum of two primes.

This is called Goldbach's Conjecture. No one has ever proved that this is true or found a counterexample to show it is false.

Example 6: Using Inductive Reasoning in Real Life

A full moon occurs when the moon is on the opposite side of Earth from the sun. During a full moon, the moon appears as a complete circle.

Use inductive reasoning and the information below to make a conjecture about how often a full moon occurs.

Specific Cases: In 2005, the first six full moons occurred on January 25, February 24, March 25, Aril 24, May 23, and June 22.

Conjecture: A full moon occurs *typically* once a month (every 29/30 days)